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Fuzzy Logic in Intelligent System Design

Theory and Applications

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Preface

We describe in this book recent advances on the use of fuzzy logic in design of hybrid intelligent systems based on nature-inspired optimization and their application in areas such as intelligent control and robotics, pattern recognition, medical diagnosis, time series prediction, and optimization of complex problems. The book is organized into nine main parts, which contain a group of papers around a similar subject. The first part consists of papers with the main theme of theoretical aspects of fuzzy logic, which basically consists of papers that propose new concepts and algorithms based on type-1 fuzzy systems. The second part contains papers with the main theme of type-2 fuzzy logic, which are basically papers dealing with new concepts and algorithms for type-2 fuzzy systems. The second part also contains papers describing applications of type-2 fuzzy systems in diverse areas, such as time series prediction and pattern recognition. The third part contains papers that present enhancements to meta-heuristics based on fuzzy logic techniques describing new nature-inspired optimization algorithms that use fuzzy dynamic adaptation of parameters. The fourth part presents emergent intelligent models, which range from quantum algorithms to cellular automata. The fifth part contains papers describing applications of fuzzy logic in diverse areas of medicine, such as diagnosis of hypertension and hearth diseases. The sixth part contains papers describing new computational intelligence algorithms and their applications in different areas of intelligent control. The seventh part contains papers that present the use of fuzzy logic in different mathematic models. The eight part deals with a diverse range of applications of fuzzy logic, ranging from environmental to autonomous navigation. The ninth part deals with theoretical concepts of fuzzy models.

In the first part of theoretical aspects of type-1 fuzzy logic, there are four papers that describe different contributions that propose new models, concepts, and algorithms centered on type-1 fuzzy systems. The aim of using fuzzy logic is to provide uncertainty management in modeling complex problems.

In the second part of type-2 fuzzy logic theory and applications, there are four papers that describe different contributions that propose new models, concepts, and algorithms centered on type-2 fuzzy systems. There are also papers that describe different contributions on the application of these kinds of type-2 fuzzy systems to solve complex real-world problems, such as time series prediction, medical diagnosis, and pattern recognition.

In the third part of fuzzy logic for the augmentation of nature-inspired optimization meta-heuristics, there are six papers that describe different contributions that propose new models and concepts, which can be considered as the basis for enhancing nature-inspired algorithms with fuzzy logic. The aim of using fuzzy logic is to provide dynamic adaptation capabilities to the optimization algorithms, and this is illustrated with the cases of the bat algorithm, harmony search, and other methods. The nature-inspired methods include variations of ant colony optimization, particle swarm optimization, the bat algorithm, as well as new nature-inspired paradigms.

In the fourth part of emergent intelligent models, there are six papers that describe different contributions on the application of these kinds of models to solve complex real-world optimization problems, such as time series prediction, robotics, and pattern recognition.

In the fifth part of fuzzy logic applications in medicine, there are three papers that describe different contributions on the application of these kinds of fuzzy logic models to solve complex real-world problems, such as medical diagnosis.

In the sixth part of intelligent control, there are six papers that describe different contributions that propose new models, concepts, and algorithms for designing intelligent controllers for different plants. The aim of using these algorithms is to provide methods and solution to some real-world problem control areas, such as scheduling, planning, and robotics.

In the seventh part, there are five papers that are presenting the application of fuzzy logic in different mathematical models. There are also papers that describe different contributions on the application of these kinds of fuzzy models to solve complex real-world problems, such as in intelligent control.

In the eighth part, there are four papers dealing with applications of fuzzy logic, like in diagnosing air quality or vehicle navigation. In addition, theoretical contributions are presented in regard to how we can apply fuzzy logic.

Finally, in the ninth part, there are six papers presenting theoretical concepts of fuzzy models. The concepts range from fuzzy linear programming to fuzzy restricted Boltzmann machines.

In conclusion, the edited book comprises papers on diverse aspects of fuzzy logic, neural networks, and nature-inspired optimization meta-heuristics and their application in areas such as intelligent control and robotics, pattern recognition, time series prediction, and optimization of complex problems. There are theoretical aspects as well as application papers.

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Can We Detect Crisp Sets Based Only on the Subsethood Ordering of Fuzzy Sets? Fuzzy Sets and/or Crisp Sets Based on Subsethood of Interval-Valued Fuzzy Sets?

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Abstract. Fuzzy sets are naturally ordered by the subsethood relation $A \subseteq B$. If we only know which set which fuzzy set is a subset of which – and have no access to the actual values of the corresponding membership functions – can we detect which fuzzy sets are crisp? In this paper, we show that this is indeed possible. We also show that if we start with interval-valued fuzzy sets, then we can similarly detect type-1 fuzzy sets and crisp sets.

1 Formulation of the Problem

Fuzzy Sets: A Brief Reminder. A *fuzzy set* is usually defined as a function $\mu : U \to [0,1]$ from some set U (called *Universe of discourse*) to the interval $[0,1]$; see, e.g., [1–3]. This function is also known as a *membership function*.

A fuzzy set A with a membership function $\mu_A(x)$ is called a *subset* of a fuzzy set B with a membership function $\mu_B(x)$ if $\mu_A(x) \le \mu_B(x)$ for all x. The subsethood relation is an *order* in the sense that it is reflexive ($A \subseteq A$), asymmetric ($A \subseteq B$ and $B \subseteq A$ imply $A = B$), and transitive ($A \subseteq B$ and $B \subseteq C$ imply $A \subseteq C$).

Traditional (*crisp*) sets S can be viewed as particular cases of fuzzy sets, with their characteristic functions playing the role of membership functions: $\mu_S(x) = 1$ if $x \in S$ and $\mu_S(x) = 0$ if $x \notin S$.

A Natural Question: Can We Detect Crisp Sets Based Only on the Subsethood Ordering of Fuzzy Sets? If we have a class F of all fuzzy sets, and for each fuzzy set A and for each element $x \in U$, we know the value $\mu_A(x)$ of the corresponding membership function, then we can easily detect which of the fuzzy sets are crisp: a fuzzy set is crisp if for every $x \in U$, we have either $\mu_A(x) = 0$ or $\mu_A(x) = 1$.

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Suppose now that we have a class F of all fuzzy sets with the subsethood ordering $A \subseteq B$ – but we have no access to the actual values of the corresponding membership functions. Based only on this ordering relation $A \subseteq B$, can we then detect crisp sets?

What If We Only Consider Interval-Valued Fuzzy Sets. A similar question can be asked if we consider interval-valued fuzzy sets, for which the value of the membership function is a subinterval of the interval $[0, 1]$: $\mu(x) = [\underline{\mu}(x), \overline{\mu}(x)] \subseteq [0, 1]$, and $A \subseteq B$ means that $\underline{\mu}_A(x) \le \underline{\mu}_B(x)$ and $\overline{\mu}_A(x) \le \overline{\mu}_B(x)$ for all x.

What We Do in This Paper. In this paper, we prove that in both cases – when we consider fuzzy sets and when we consider interval-valued fuzzy sets – we can indeed detect crisp sets and type-1 fuzzy sets based only on the subsethood relation $A \subseteq B$.

2 What If We Consider [0, 1]-Based Fuzzy Sets

Our Plan. To describe crisp sets in terms of the subsethood relation $A \subseteq B$, we will follow the following four steps:

- first, we will prove that the empty set $\emptyset$ can be uniquely determined based on the subsethood relation;
- second, we will show that 1-element crisp sets, i.e., sets of the type $\{x_0\}$, can be thus determined,
- third, we will prove that 1-element fuzzy sets, i.e., fuzzy sets A for which for some $x_0 \in U$, we have $\mu_A(x_0) > 0$ and $\mu_A(x) = 0$ for all $x \neq x_0$, can be determined based on the subsethood relation, and
- finally, we prove that crisp sets can be uniquely determined based on the subsethood relation.

First Step: How to Detect an Empty Set? An empty set $\emptyset$ is a fuzzy set for which $\mu_\emptyset(x) = 0$ for all $x \in U$. The detection of an empty set can be made based on the following simple result:

Proposition 1. *A fuzzy set A is an empty set if and only if $A \subseteq B$ for all fuzzy sets B.*

Proof.
1°. Let us first prove that when $A = \emptyset$, then $A \subseteq B$ for all fuzzy sets B.

Indeed, for every fuzzy set B, we have $0 \le \mu_B(x)$ for all x and thus, $\mu_\emptyset(x) = 0 \le \mu_B(x)$ for all x, i.e., we indeed have $\emptyset \subseteq B$.

2°. Let us now prove that, vice versa, if for some fuzzy set A, we have $A \subseteq B$ for every possible fuzzy set B, then $A = \emptyset$.

Indeed, in particular, the property $A \subseteq B$ is true for the case when B is the empty set. In this case, from the fact that $\mu_A(x) \le \mu_B(x) = \mu_\emptyset(x) = 0$, we conclude that $\mu_A(x) = 0$ for all x, i.e., that A is indeed the empty set.

The proposition is proven.

Second Step: How to Detect 1-element Crisp Sets Based on the Subsethood Relation. Let us prove the following auxiliary result.

Proposition 2. *A non-empty fuzzy set A is a one-element crisp set if and only if the following two conditions are satisfied:*

- *the class $\{B : B \subseteq A\}$ is linearly ordered and*
- *for no proper superset A' of A, the class $\{B : B \subseteq A'\}$ is linearly ordered.*

Proof.
1°. Let us first prove that every 1-element crisp set, i.e., every set of the type $A = \{x_0\}$, satisfies the above two properties.

1.1°. Let us prove the first property: that the class $\{B : B \subseteq A\}$ is linearly ordered.

Indeed, for the given set A, we have $\mu_A(x_0) = 1$ and $\mu_A(x) = 0$ for all $x \neq x_0$. So, if $B \subseteq A$, i.e., if $\mu_B(x) \leq \mu_A(x)$ for all x, this means that $\mu_B(x) = 0$ for all $x \neq x_0$. Thus, for such sets B, the only non-zero value of the membership function may be attained when $x = x_0$.

So, if we have two sets $B \subseteq A$ and $B' \subseteq A$, then for these two sets, $\mu_B(x) = \mu_{B'}(x) = 0$ for all $x \neq x_0$. Thus:

- if $\mu_B(x_0) \leq \mu_{B'}(x_0)$, then, as one can easily check, we have $\mu_B(x) \leq \mu_{B'}(x)$ for all x, i.e. we have $B \subseteq B'$, and
- if $\mu_{B'}(x_0) \leq \mu_B(x_0)$, then, as one can easily check, we have $\mu_{B'}(x) \leq \mu_B(x)$ for all x, we have $B' \subseteq B$.

Thus, for every two fuzzy sets B and B' from the class $\{B : B \subseteq A\}$, we have either $B \subseteq B'$ or $B' \subseteq B$. So, this class is indeed linearly ordered.

1.2°. Let us now prove that no proper superset A' of the 1-element set $A = \{x_0\}$ has the property that the class $\{B : B \subseteq A'\}$ is linearly ordered.

For the set $A = \{x_0\}$, we have $\mu_A(x_0) = 1$ and $\mu_A(x) = 0$ for all other x. If A' is a superset of A, this means that $\mu_{A'}(x) = 1$. The fact that A' is a proper superset means that $A' \neq A$, thus we have $\mu_{A'}(x') > 0$ for some $x' \neq x_0$. In this case, we can define the following fuzzy set B: $\mu_B(x') = \mu_{A'}(x')$ and $\mu_B(x) = 0$ for all $x \neq x_0$. Then, we have $B \subseteq A'$, $A \subseteq A'$, but $B \not\subseteq A$ (since $\mu_B(x') > 0$ and thus, $\mu_B(x') \not\leq \mu_A(x') = 0$) and $A \not\subseteq B$ (since $1 = \mu_A(x_0) \not\leq \mu_B(x_0) = 0$). Thus, the class $\{B : B \subseteq A'\}$ is indeed not linearly ordered.

2°. Let us prove that, vice versa, if a fuzzy set A has the above two properties, then it is a one-element crisp set.

2.1°. Let us first prove, by contradiction, that we can only have one element x for which $\mu_A(x) > 0$. Indeed. if $\mu_A(x_1) > 0$ and $\mu_A(x_2) > 0$ for some $x_1 \neq x_2$, then we can take the following fuzzy sets B_1 and B_2:

- $\mu_{B_1}(x_1) = \mu_A(x_1)$ and $\mu_{B_1}(x) = 0$ for all other x, and
- $\mu_{B_2}(x_2) = \mu_A(x_2)$ and $\mu_{B_2}(x) = 0$ for all other x.

Here, $B_1 \subseteq A$ and $B_2 \subseteq A$, but $B_2 \not\subseteq B_2$ and $B_2 \not\subseteq B_1$ – which contradicts to our assumption that the class $\{B : B \subseteq A\}$ is linearly ordered.

2.2°. Due to Part 2.1, we have $\mu_A(x_0) > 0$ for at most one element x_0; for all $x \neq x_0$, we have $\mu_A(x) = 0$. Let us prove, by contradiction, that $\mu_A(x_0) = 1$, i.e., that A is indeed a one-element crisp set.

Indeed, if $\mu_A(x_0) < 1$, then we can consider the following proper superset $A' \supseteq A$: $\mu_{A'}(x_0) = (1 + \mu_A(x_0))/2 < 1$ and $\mu'_A(x) = 0$ for all other x. Similarly to Part 1.1 of this proof, we can prove that for this superset A', the class $\{B : B \subseteq A'\}$ is linearly ordered – which contradicts to our assumption that such a proper superset does not exist.

The proposition is proven.

Third Step: How to Detect 1-element Fuzzy Sets Based on the Subsethood Relation. We say that a fuzzy set is a *1-element set* if for some $x_0 \in X$, we have $\mu_A(x_0) > 0$ and $\mu_A(x) = 0$ for all $x \neq x_0$. Let us prove the following auxiliary result.

Proposition 3. *A non-empty fuzzy set A is a one-element fuzzy set if and only if the class $\{B : B \subseteq A\}$ is linearly ordered.*

Proof.
1°. Arguments similar to Part 1.1 of the proof of Proposition 2 show that if A is a one-element fuzzy set, then the class $\{B : B \subseteq A\}$ is linearly ordered.

2°. Vice versa, if A is not an empty set and not a one-element fuzzy set, this means that there exist at least two values $x_1 \neq x_2$ for which $\mu_A(x_1) > 0$ and $\mu_A(x_2) > 0$. We can then take the following fuzzy sets B_1 and B_2:

- $\mu_{B_1}(x_1) = \mu_A(x_1)$ and $\mu_{B_1}(x) = 0$ for all $x \neq x_1$, and
- $\mu_{B_2}(x_2) = \mu_A(x_2)$ and $\mu_{B_2}(x) = 0$ for all $x \neq x_2$.

Then $B_1 \subseteq A$ and $B_2 \subseteq A$, but $B_1 \not\subseteq B_2$ and $B_2 \not\subseteq B_1$. Thus, the class $\{B : B \subseteq A\}$ is not linearly ordered.

The proposition is proven.

Final Result: How to Detect Crisp Sets Based on the Subsethood Relation. Let us prove the following auxiliary result.

Theorem 1. *A fuzzy set A is crisp if and only if every one-element fuzzy subset $B \subseteq A$ can be embedded in a one-element crisp subset of A.*

Comment. In other words,

$$A \text{ is crisp} \Leftrightarrow \forall A\,(B \text{ is a one-element fuzzy subset of } A \Rightarrow$$

$$\exists C\,((B \subseteq C \subseteq A)\,\&\,(C \text{ is a 1-element crisp set}))).$$

Proof.
1°. Let A be a crisp set, and let $B \subseteq A$ be a 1-element fuzzy set. By definition, this means that for some x_0, we have $\mu_B(x_0) > 0$ and $\mu_B(x) = 0$ for all other x.

Since the set A is crisp, the only possible values of $\mu_A(x_0)$ are 0 and 1. From $\mu_B(x_0) \le \mu_A(x_0)$, we conclude that $\mu_A(x_0) > 0$ and thus, that $\mu_A(x_0) = 1$. So, $x_0 \in A$ and hence $B \subseteq \{x_0\} \subseteq A$.

2°. Vice versa, if A is not a crisp set, this means that for some element x_0, we have $0 < \mu_A(x_0) < 1$. In this case, we can take the following 1-element fuzzy set $B \subseteq A$: $\mu_B(x_0) = \mu_A(x_0)$ and $\mu_B(x) = 0$ for all $x \ne x_0$. Here, $B \subseteq A$, but the only 1-element crisp set C containing B is the set $C = \{x_0\}$, and this 1-element crisp set is *not* a subset of the original set A: $C \not\subseteq A$.

The theorem is proven.

3 What If We Consider Interval-Valued Fuzzy Sets

First Step: How to Detect an Empty Set. An empty set $\emptyset$ is an interval-valued fuzzy set for which $\mu_\emptyset(x) = [0, 0]$ for all $x \in U$. The detection of an empty set can be made based on the following result:

Proposition 4. *An interval-valued fuzzy set A is an empty set if and only if $A \subseteq B$ for all interval-valued fuzzy sets B.*

Proof is similar to proof of Proposition 1.

Second Step: How to Detect Special 1-element Interval-Valued Fuzzy Sets Based on the Subsethood Relation. Let's introduce an auxiliary notion. We say that an interval-valued fuzzy set A is *special* if for some element x_0, we have $\mu_A(x_0) = [0, a]$ for some number $a > 0$ and $\mu_A(x) = 0$ for all $x \ne x_0$.

Proposition 5. *A non-empty interval-valued fuzzy set A is special if and only if the class $\{B : B \subseteq A\}$ is linearly ordered.*

Proof.
1°. For special sets (in the sense of the above definition), the fact that the class $\{B : B \subseteq A\}$ is linearly ordered can be proven similarly to Part 1.1 of the proof of Proposition 2.

2°. Let us now prove that, vice versa, if for some non-empty interval-valued fuzzy set A, the class $\{B : B \subseteq A\}$ is linearly ordered, then the set A is special.

2.1°. Since A is non-empty, there exists an element x_0 for which $\mu_A(x_0) \ne [0, 0]$. Let us prove, by contradiction, that for every other element $x \ne x_0$, we have $\mu_A(x) = [0, 0]$.

Indeed, if we had $\mu_A(x_1) \ne [0, 0]$ for some $x_1 \ne x_0$, then we would be able to take the following two sets B_0 and B_1:

- $\mu_{B_0}(x_0) = \mu_A(x_0)$ and $\mu_{B_0}(x) = [0, 0]$ for all $x \neq x_0$, and
- $\mu_{B_1}(x_1) = \mu_A(x_1)$ and $\mu_{B_1}(x) = [0, 0]$ for all $x \neq x_1$.

In this case, $B_0 \subseteq A$ and $B_1 \subseteq A$, but $B_0 \not\subseteq B_1$ and $B_1 \not\subseteq B_0$. This contradicts our assumption that the class $\{B : B \subseteq A\}$ is linearly ordered.
2.2°. To complete the proof of the proposition, we need to prove that the value $\mu_A(x_0) = [\underline{\mu}_A(x_0), \overline{\mu}_A(x_0)]$ has the form $[0, a]$ for some $a > 0$, i.e., that $\underline{\mu}_A(x_0) = 0$.

We will prove it by contradiction. Suppose that, vice versa, $\underline{\mu}_A(x_0) > 0$. In this case, we can take the following sets B_1 and B_2:

- $\mu_{B_1}(x_0) = [0.5 \cdot \underline{\mu}_A(x_0), 0.5 \cdot \underline{\mu}_A(x_0)]$ and $\mu_{B_1}(x) = 0$ for all $x \neq x_0$, and
- $\mu_{B_2}(x_0) = [0, \underline{\mu}_A(x_0)]$ and $\mu_{B_2}(x) = 0$ for all $x \neq x_0$.

Then, $B_1 \subseteq A$ and $B_2 \subseteq A$, but $B_1 \not\subseteq B_2$ and $B_2 \not\subseteq B_1$. This contradicts our assumption that the class $\{B : B \subseteq A\}$ is linearly ordered.

The proposition is proven.

Third Step: How to Detect 1-element type-1 Fuzzy Sets Based on the Subsethood Relation. We say that an interval-valued fuzzy set is a *1-element type-1* fuzzy set if there exists an element x_0 for which $\mu_A(x_0) = [a, a]$ for some $a > 0$ and $\mu_A(x) = [0, 0]$ for all $x \neq x_0$.

Proposition 6. *A non-empty interval-valued fuzzy set A is a 1-element type-1 set if and only if it is satisfies the following three properties:*

- *the set A is not special (in the sense of the above definition),*
- *there exists a special set $B \subseteq A$ for which the class $\{C : B \subseteq C \subseteq A\}$ is linearly ordered, and*
- *for no proper superset A' of A, the class $\{C : B \subseteq C \subseteq A\}$ is linearly ordered.*

Proof is similar to the proof of Proposition 2.

Final Result. Since we have subsethood, we also have union: the union of A_α is the $\subseteq$-smallest set that contains all $A - \alpha$. We can thus define type-1 fuzzy sets as unions of 1-element type-1 fuzzy sets. Once we can detect type-1 fuzzy sets, we can use techniques from the previous section to detect crisp sets. Thus, *we can indeed detect type-1 fuzzy sets and crisp sets based only on subsethood relation between interval-valued fuzzy sets.*

References

1. Klir, G., Yuan, B.: Fuzzy Sets and Fuzzy Logic. Prentice Hall, Upper Saddle River (1995)
2. Nguyen, H.T., Walker, E.A.: A First Course in Fuzzy Logic. Chapman and Hall/CRC, Boca Raton (2006)
3. Zadeh, L.A.: Fuzzy sets. Inf. Control **8**, 338–353 (1965)