Towards a More Realistic Treatment of Uncertainty in Earth and Environmental Sciences: Beyond a Simplified Subdivision into Interval and Random Components

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When processing data, it is often very important to take into account measurement uncertainty, i.e., the fact that the measurement results $\widetilde{x}$ are, in general, different from the actual (unknown) value x of the corresponding quantity. In measurement theory, traditionally, a measurement error $\Delta x \stackrel{\text{def}}{=} \widetilde{x} - x$ is subdivided into random and systematic components $\Delta x = \Delta_s x + \Delta_r x$ (see, e.g., [2]): the systematic error component $\Delta_s x$ is usually defined as the expected value $\Delta_s x = E[\Delta x]$, while the random error component is usually defined as the difference $\Delta_r x \stackrel{\text{def}}{=} \Delta x - \Delta_s x$. By definition, the systematic error component does not change from measurement to measurement, while the random errors $\Delta_r x$ corresponding to different measurements are usually assumed to be independent.

For the systematic error component, we only know the upper bound Δ_s for which $|\Delta_s x| \le \Delta_s$. Thus, the only information that we have about the value of this component is that it belongs to the interval $[-\Delta_s, \Delta_s]$. Because of this fact, interval computations are used for processing the systematic errors. The random error component is usually characterized by the corresponding probability distribution; often, it is assumed to be Gaussian, with a known standard deviation σ.

For many Earth and environmental science measurements, the differences $\Delta_r x = \Delta x - \Delta_s x$ corresponding to nearby moments of time are often strongly correlated. For example, meteorological sensors may have daytime or nighttime biases, or winter and summer biases. To capture this correlation, environmental science researchers proposed an empirically successful semi-heuristic three-component model of measurement error. In this model, the difference $\Delta x - \Delta_s x$ is represented as a combination of a "truly random" error $\Delta_t x$ (which is independent from one measurement to another), and a new "periodic" component $\Delta_p x$.

We provide a theoretical explanation for this heuristic three-component model, and we show how to extend the traditional interval and probabilistic error propagation techniques to this three-component model. Our preliminary results are described in [3].

In practice, instead of a *single* quantity x (temperature, density, etc.), we often have a *field* $x(s)$ in which the value of the quantity changes with a spatial location s (and, sometimes, with time t). For fields, the measurement error $\widetilde{x}(s) - x(s)$ is caused not only by the inaccuracy of the measuring instrument (MI), but also by the fact that the output $\widetilde{x}(s)$ of the MI is determined by the *average* $\int K(s - s') \cdot x(s')\, ds'$ over a neighborhood $s' \approx s$ (here, $K(s)$ describes the instrument's *spatial resolution*). In the talk, we describe how to take into account this additional uncertainty, and how to decrease it by merging ("fusing") two results $\widetilde{x}_1(s)$ and $\widetilde{x}_2(s)$ obtained from measuring the same field $x(s)$; our preliminary results appeared in [1]. As a case study, we consider the combination of density descriptions obtained from seismic measurements and from gravity measurements.

References:

[1] O. Ochoa, A. A. Velasco, C. Servin, and V. Kreinovich, "Model Fusion under Probabilistic and lnterval Uncertainty, with Application to Earth Sciences", lnternational Journal of Reliability and Safety, 2012, Vol. 6, No. 1–3, pp. 167–187.

[2] S. Rabinovich, *Measurement Errors and Uncertainties: Theory and Practice*, American Institute of Physics, New York, 2005.

[3] C. Servin, M. Ceberio, A. Jaimes, C. Tweedie, and V. Kreinovich, *How to Describe and Propagate Uncertainty When Processing Time Series*, Univ. of Texas at El Paso, Dept. Computer Science, Technical Report UTEP-CS-12-01a, 2012, http://www.cs.utep.edu/vladik/2012/tr12-01a.pdf